Comparing HDI and Per Capita Electricity Use Data with an Improved Hyperbolic Tangent Correlation

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15 April 2006 GNEP_MG_R_04

Abstract

The formula HDI = tanh[(u/u_0)^(1/4)], with u in kWh and u_0 = 1400 kWh, correlates the national data for HDI and per capita electricity use as reported by the United Nations Development Programme for 2005. This formula duplicates the shape of the Pasternak correlation over the range of the data, and it meets the limits (u, HDI) = (0, 0) and (infinity, 1). There is no "law" or necessity that this formula relate HDI and u, it is simply an observation that it fits the trend of the data. The "proper" HDI for any given u is a matter of speculation, to be inferred by deeper study of how the availability of an energy technology "causes" socio-economic development.

Previous Correlation

Pasternak correlated HDI and E [national annual electricity use per capita in kilowatt-hours (kWh)] in 2000, using data for 1997. He used a logarithmic form to correlate data from 60 countries. This form is not ideal because indicated HDI plunges to negative infinity at zero energy, and indicated HDI grows larger than 1 at large energy. (1)

A hyperbolic tangent meets the limiting cases exactly, and it has a "knee" in HDI between 0.3 and 0.9, followed by an asymptote to 1 at large energy. (2)

However, one must compare with actual data to find the best correlation. This report describes that comparison. The result is an improvement to the hyperbolic tangent formula. The correlation formula is

HDI = tanh[(u/u_0)^(1/4)],

u_0 = 1400 kWh,

with u in kWh. Why the quarter root of the energy ratio?

Comparing Correlations

Figure 1 shows the improved hyperbolic tangent form, called H4, and Pasternak's logarithmic form, called P, both plotted against the energy use parameter u (kWh/c).

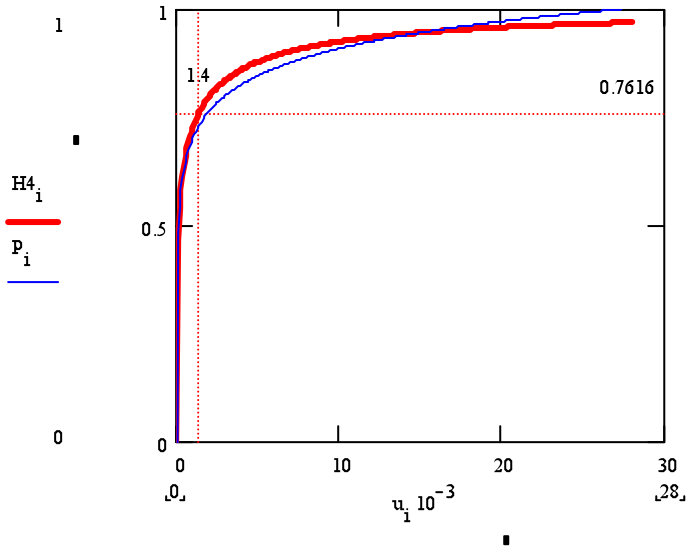

Figure 1, H4 (bold solid) and Pasternak (thin sold) Correlations to 28 kWh/c

A hyperbolic tangent form that conforms more tightly to Pasternak's curve can be had by adjusting the power on the energy ratio (here it is ¼) and the normalization energy u_0 (here it is 1400 kWh/c). However, the H4 curve seems to suit the data, as will be shown.

"H form" Curves

Figure 2 shows the effect of the power on the energy ratio. As the fractional power of the energy ratio diminishes, the hyperbolic tangent form is raised for $u < u_0$, and it is depressed for $u > u_0$; but all hyperbolic tangent forms meet at u = 0 (HDI = 0) and u = infinity (HDI = 1).

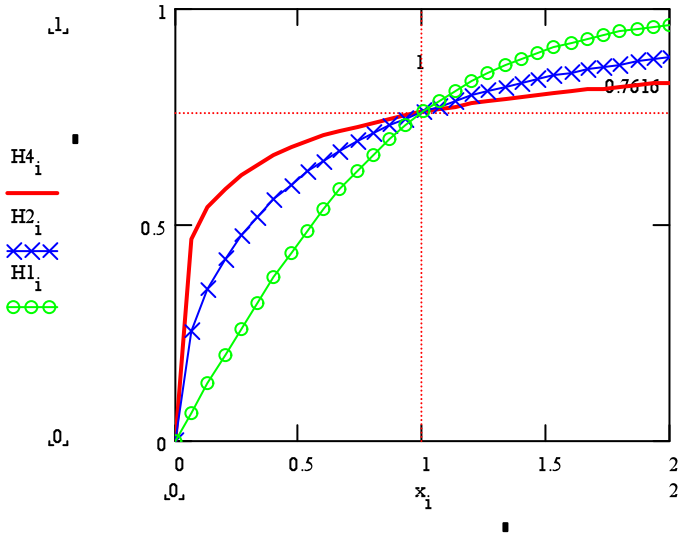

Figure 2, H Forms, Energy Ratio Powers: H4 @ ¼ (bold), H2 @ ½ (x), H1 @ 1 (0)

Data and Correlation

Figure 3 shows the data for 2005 (177 nations) and the H4 correlation; Figure 4 repeats the display but on a logarithmic scale of energy. (3), (4), (5)

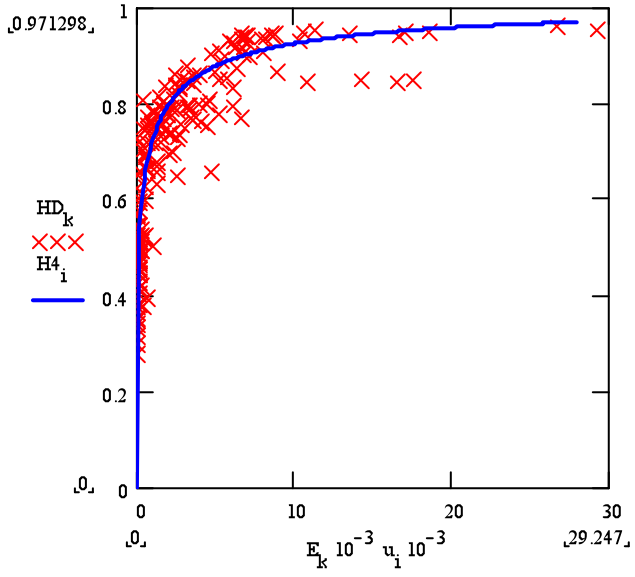

Figure 3, Data (E, HD) and H4 Correlation to 29 kWh/c

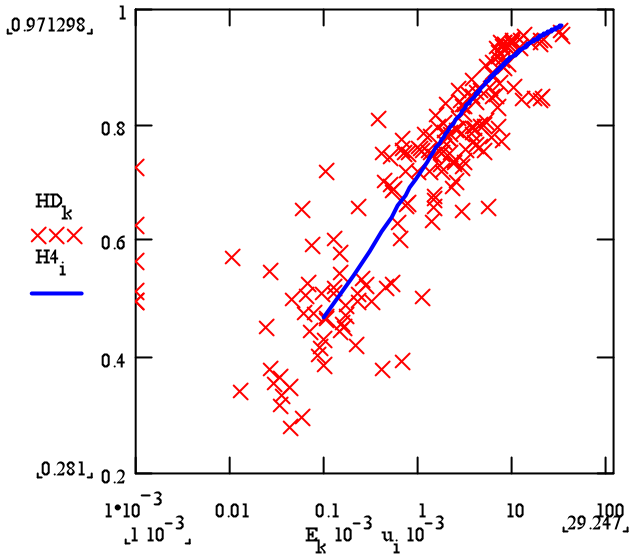

Figure 4, HDI vs. Log of Electricity Use (kWh/c), Data and H4 Correlation [reported E spans from 10 kWh/c to 29,247 kWh/c]

Countries without reported values of electricity use have been assigned 1 kWh/c. This removes them from the energy cluster but preserves the listing and ranking of HDI. One might infer an electricity use for these countries from their reported HDI by the H4 formula.

The final pair of displays are linear and semi-log plots of the data and the inferred HDI from the reported electricity use E. To paraphrase, the H4 "expectation" for each nation from its recorded E is shown along with the actual data.

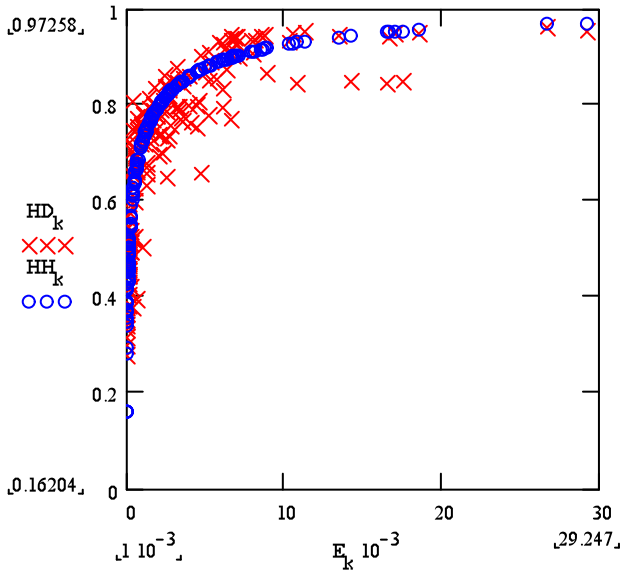

Figure 5, HDI & E Data (x), and H4 Expectation based on E Data (0)

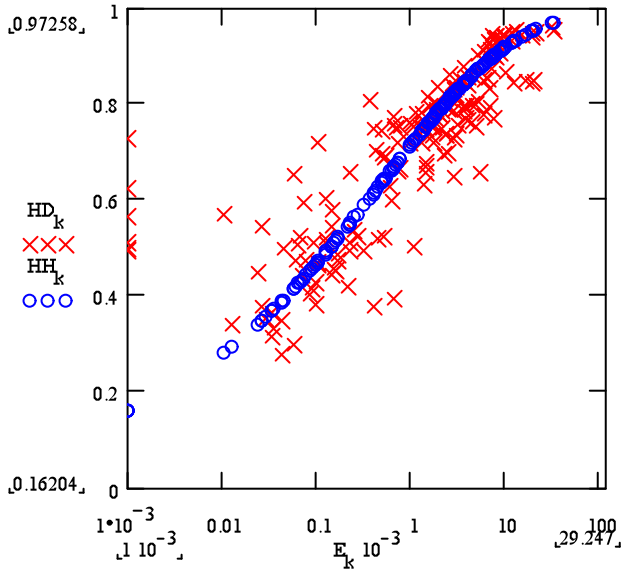

Figure 6, HDI & Log(E) Data (x), and H4 Expectation based on Log(E) Data (0)

References

[1] Alan D. Pasternak, “Global Energy Futures and Human Development: A Framework for Analysis,” *Lawrence Livermore National Laboratory* (USDOE), October 2000, UCRL-ID-140773, http://www.llnl.gov/tid/lof/documents/pdf/239193.pdf, (6 April 2006)

[2] Manuel Garcia, "A Hyperbolic Tangent Correlation Between Human Development and Electricity Use," *Lawrence Livermore National Laboratory/Engineering* (USDOE), 9 April 2006, GNEP_MG_R_03

[3] Human Development Report for 2005, *United Nations,* http://hdr.undp.org/reports/global/2005/pdf/HDR05_HDI.pdf, (14 April 2006)

Human Development Reports, Get Data http://hdr.undp.org/statistics/data/, (14 April 2006)

Human Development Index (2003, as included in HDR2005) http://hdr.undp.org/statistics/data/indicators.cfm?x=1&y=1&z=1, (14 April 2006)

[4] List of countries by Human Development Index, http://en.wikipedia.org/wiki/List_of_countries_by_Human_Development_Index , "This is a list of countries by Human Development Index (2003), as included in the United Nations Development Programme Report 2005," (14 April 2006)

[5] Electricity Consumption Per Capita (UNDR 2005), *United Nations Development Programme*, http://hdr.undp.org/statistics/data/indicators.cfm?x=206&y=1&z=1, (14 April 2006)