The 2D electric field above a planar sequence of independent strip electrodes

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This paper describes an analytical solution for the electrostatic potential and electric field in dielectrophoretic (DEP) microfluidic devices. DEP devices are engineered to be DNA concentrators in automated, flow-through systems of sample preparation for biological assays. DEP devices deflect and trap particles suspended in liquid flows because macromolecular particles polarize in response to an electric field, and the coupling of this polarization with the field alters particle motions. The DEP effect is proportional to $\nabla(\mathbf{E}\cdot\mathbf{E})$. The electric field is produced by a sequence of independent strip electrodes, and is two dimensional throughout the volume of the flow. As the potentials used in DEP devices are low frequency sinusoids, the field of interest is the gradient of the squared root-mean-square (rms) of the electric field, $\nabla E_{\mathrm{rms}}^2$. An example of this field is shown.

Defining the problem

Improving both the sensitivity and speed of tests, which detect specific DNA sequences in fluid samples, is required to further advance biomedical research. Automating the preparation of samples for biological assays would reduce waste and cost, while speeding results. This paper describes an electrical aspect of one particular technology, dielectrophoretic (DEP) microfluidic devices, aiming at such automation.

The presence of target macromolecular particles in a fluid sample may be enhanced physically by concentration with a centrifuge, or chemically by purification through an involved recipe specific to the target chemical. It is cumbersome to include these procedures in an automated sample preparation system. A new approach is to concentrate biological particles by straining them with an electric field as fluid flows over a segmented electrode plane. This plane has a sequence of parallel electrode strips, with intervening dielectric spaces, all perpendicular to the direction of flow. Biological particles can be drawn from the flow and collected in regions of high field strength by a dielectrophoretic force. Activating the field for a period of time creates a higher concentration of target material above the electrode array. Deactivating the field causes this slug of enhanced concentration to flow downstream to a collection site. **Figure 1** is a schematic of a DEP microdevice.

A biological particle becomes polarized in the presence of an electric field, and the coupling of this polarization with the field alters the motion of the particle. This dielectrophoretic effect is one example of an electrokinetic phenomena, see [1] and [2]. The dielectrophoretic force is proportional to the square of the electric field because it combines two factors each linear in field strength: the polarization of the particle and the electric force on a charge. See [3] for a detailed presentation on dielectrophoresis. Polarized particles are attracted to regions of either high or low field gradient magnitude, depending on the differences in electrical properties between the particles and the suspending medium. This force can be generated with both AC and DC fields, though AC is typical because the frequency can then be selected to optimize the effect, see [4]. Concentrating biological particles by the DEP force becomes practical in microfluidic devices because sufficient field strength is developed with only a few volts, and the volume of flow may only span a centimeter of length with a cross section less than a square millimeter, see [5] and [6].

One aspect of refining this technology is the use of computer modeling to improve the design of DEP microdevices by calculating the effect of electrical, flow, and geometry parameters on the trajectories of target biological particles. One aspect of the theoretical work, which enables computer simulations, is to improve the description of the electrostatic field and to simplify its calculation, see [7] for extensive work in this regard. An ideal result from theory would be formulas, which could easily be embedded as a subroutine in any device simulation code, for the electric field at any point and

time in a DEP microdevice. This paper presents such results. The formulas presented here have already been incorporated as a subroutine into a computer simulation of prototype DEP microdevices similar to those described in [6]. This paper presents an analytical solution of the DEP microdevice electrostatic problem, which is both more general and accurate than the procedure described in [7], and which requires much less time to calculate. The remainder of this paper describes the analytical solution in as much detail as possible.

The analytical solution presented here is based on a conformal mapping between cartesian and cylindrical coordinates. The potential of the strip array is a sum of the potentials from each electrode strip, which are all assumed independent. Each strip electrode in cartesian coordinates is mapped onto a cylindrical electrode in cylindrical coordinates. The potential for the cylinder array is a sum of potentials each produced by a line source within each cylinder. The potential and field for the cylinder array are mapped back to the strip array to complete the solution. The AC DEP force is proportional to the gradient of the square of the root-mean-square of the electric field, $\nabla E_{\mathrm{rms}}^2$, see [3]. The arrays we are interested in may have an arbitrary sequence of strip voltages, but all share a single excitation frequency.

This report will proceed through a sequence of topics as follows: the coordinate systems required, the conformal mapping and its reverse, the cylindrical potential for a line source, the potential of the cylinder array, the potential of the strip array, the electric field of the strip array, the effect of the second boundary condition on the array potential, the gradient of E·E, and rms quantities for a single frequency array.

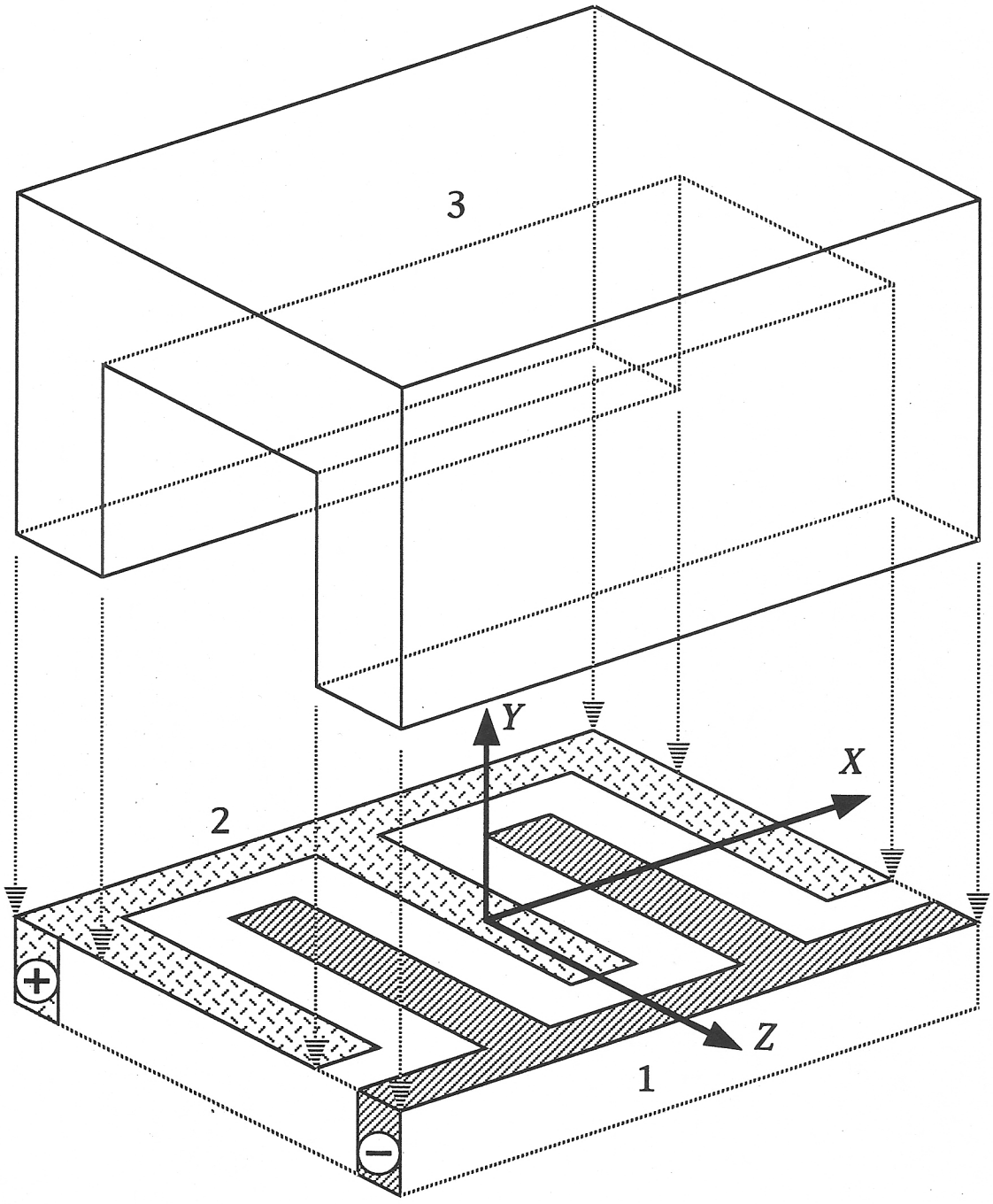

Figure 1, Schematic of dielectrophoretic microdevice

Figure captions

1 Schematic of dielectrophoretic microdevice. The origin of the global coordinate system is at the center of the electrode array, which is in the *XZ* plane, with flow parallel to X, and with the electric field intensity decreasing along Y. Item 1 is a glass substrate, typically millimeters thick (along Y), and up to several centimeters wide (along Z) and long (along X). Items 2 are thin electrode structures. A bipolar arrangement of interdigitated electrode strips is shown, however a sequence of independently activated electrode strips is possible. Typical electrode strips are 30 μm wide (along X) with a similar distance for the intervening dielectric spaces. Arrays can have up to 50 strips. Item 3 is a block containing the flow channel, which may be millimeters to a centimeter in length (along X), hundreds of microns to a millimeter in width (along Z), and a height (along Y) of hundreds of microns. The channel height is large compared to the electrode width (along X) and spacing to ensure that particles sense the net field from many electrode strips during their flow over the array. Biological particles may be several μm in diameter. This schematic is not to scale. Also, the flow channel is actually carved into the substrate and then electrodes are vapor-deposited over the top of the substrate; item 3 is then a simple cover plate. This schematic describes the geometry of the electrostatic problem more clearly than a mechanically realistic drawing.