No emissions with exponential decay of CO_2 concentration Model

18 June 2020

M. Garcia, Jr

Assume that at time (year) t_0 all CO_2/greenhouse gas emission stop; and also that the existing atmospheric concentration of greenhouse gases (mainly CO_2) decays exponentially with time constant μ. That decay is due first to absorption by the oceans (a 20-200 year process) and then (since the oceans saturate with CO_2, acidifying and killing most sea life) by the weathering of rocks (a 100,000 year to 200,000 year process, as during the PETM).

What is the resulting time history?

Using the previous model (example #5) we say that:

t_ϕ = a year (after 2020) relative to 1910
$\therefore \geq 110$ years. (relative time)

$T(t_\phi)$ = given by previous model. Note "T" is alway relative to the "T_0," which was the temperature in 1910. Mathematically, it is easiest to consider all T the be <u>temperature differences</u> relative to 1910 (when the temperature difference was zero, by definition).

$A(t_\phi)$ = albedo at $t = t_\phi$, which is:

$$= A_0 + a_{cloud}\, T(t_\phi) - a_{ice}\, T(t_\phi) + A_{pollution}\, t_\phi$$

remember these are differences

$$= A_0 + \underbrace{(a_{cloud} - a_{ice})}_{a_1}\, T(t_\phi) + A_{pollution}\, t_\phi$$

as in example #5

$F(t_\phi)$ = infrared/heat absorptivity at $t = t_\phi$

$$F(t_\phi) = \underset{(1910)}{F_0} + f_{pollution}\, t_\phi$$

Let $F(t_\phi) \equiv F_1$ (just a simpler symbol)

$$\therefore \quad F_1 = \underset{(1910)}{F_0} + f_{pollution}\, t_\phi$$

↳ time since 1910

Now, for $t \geq t_\phi$ (the gg-free future)

the infrared/heat absorptivity <u>decays</u>:

$$F(t_2 \geq t_\phi) = F_0 + (F_1 - F_0)\, e^{-\mu(t_2 - t_\phi)}$$

$$\left.\begin{aligned} T_1 &= T(t_\phi) \\ A_1 &= A_0 + a_1 T_1 + A_{pollution}\, t_\phi \\ F_1 &= F_0 + f_{pollution}\, t_\phi \end{aligned}\right\} \text{all found from example \#5 @ } t_0 \ (>1910)$$

$$\boxed{\text{These are the initial conditions to the post-emission, heat absorptivity exponential decay mode of } T \text{ vs } t \text{ for } \underline{\text{after}}\ t_0}$$

$$A(t_2 > t_1) = A_1 + a_1(T_2 - T_1) \quad , \quad \underline{A_{pollution} \to 0}$$

The post to system is:

$$C\frac{d(T_2-T_1)}{d(t_2-t_1)} = S'(1-A)-Q'(1-F)$$

$A = A_1 + a_1(T_2-T_1)$; $A_{pollution} \rightarrow 0$ quickly, rain-out.

$F = F_0 + (F_1-F_0)e^{-u(t_2-t_0)}$; $F_0 \rightarrow$ as in previous model / example #5

$t_2 \geq t_0$ (which is $\geq$ 110 years after 1910)

Again, S' and Q' are kept constant because the temperature differences to be encounter are to small to alter them significantly.

Initial (@ $t_2 = t_0$) conditions:

$T(t_0) = T_1$

$A(t_0) = A_1$

$F(t_0) = F_1$

$$T_2 - T_1 \equiv H$$
$$t_2 - t_1 \equiv \tau$$

$$C \frac{dH}{d\tau} = S'(1-A) - Q'(1-F)$$

$$A = A_1 + a_1 H$$
$$F = F_0 + (F_1 - F_0) e^{-\mu \tau}$$

$$H(\tau = 0) = 0$$
$$A(\tau = 0) = A_1$$
$$F(\tau = 0) = F_1$$

$$\therefore \frac{dH}{d\tau} = \frac{S'}{C}(1-A) - \frac{Q}{C}(1-F)$$

$$\left.\begin{aligned} \sigma &\equiv S'/C \\ \chi &\equiv Q'/C \end{aligned}\right\} \text{ as before}$$

$$\frac{dH}{d\tau} = \sigma(1-A) - \chi(1-F)$$
$$= \sigma(1 - A_1 - a_1 H) +$$
$$- \chi\left(1 - F_0 - (F_1 - F_0) e^{-\mu \tau}\right)$$

$$\frac{dH}{d\tau} = \sigma(1-A_1) - \sigma a_1 H + -\chi(1-F_0) + \chi(F_1-F_0)e^{-\mu\tau}$$

$$\frac{dH}{d\tau} = \left\{\sigma(1-A_1) - \chi(1-F_0)\right\} - \sigma a_1 H + \chi(F_1-F_0)e^{-\mu\tau}$$

Let $\underset{(\text{new } \alpha)}{\alpha_1} = \left\{\sigma(1-A_1) - \chi(1-F_0)\right\}$

$\beta = \sigma a_1$ (same β)

$\underset{(\text{new } \gamma)}{\gamma_1} = \chi[F_1 - F_0]$

$$\therefore \boxed{\frac{dH}{d\tau} = \alpha_1 - \beta H + \gamma_1 e^{-\mu\tau}}$$

$\tau \geq 0$ $\boxed{\mu \geq 0}$

$H(\tau=0) = 0$

$\alpha_1 \rightarrow$ could be $+, -, 0$

$\beta \rightarrow > 0$ $(a_1 > 0)$

$\gamma_1 \rightarrow > 0$

$$\frac{dH}{d\tau} + \beta H = \alpha_1 + \gamma_1 e^{-\mu\tau}$$

$$e^{-\beta\tau}\frac{d}{d\tau}\left[e^{\beta\tau}H\right] = \alpha_1 + \gamma_1 e^{-\mu\tau}$$

$$\frac{d}{d\tau}\left[e^{\beta\tau}H\right] = \alpha_1 e^{\beta\tau} + \gamma_1 e^{(\beta-\mu)\tau}$$

$$\int_0^\tau \frac{d}{d\tau}\left[e^{\beta\tau}H\right]d\tau = \alpha_1\int_0^\tau e^{\beta\tau}d\tau + \gamma_1\int_0^\tau e^{(\beta-\mu)\tau}d\tau$$

$$e^{\beta\tau}H - \cancel{e^{0}H(0)}^{\,0} = \frac{\alpha_1}{\beta}\left[e^{\beta\tau}-1\right] + \gamma_1\int_0^\tau e^{+(\beta-\mu)\tau}d\tau$$

$$e^{\beta\tau}H = \frac{\alpha_1}{\beta}\left[e^{\beta\tau}-1\right] + \gamma_1\int_0^\tau e^{(\beta-\mu)\tau}d\tau$$

$$H = \frac{\alpha_1}{\beta}\left[1-e^{-\beta\tau}\right] + \gamma_1 e^{-\beta\tau}\int_0^\tau e^{(\beta-\mu)\tau}d\tau$$

Let $h = \gamma_1 e^{-\beta\tau}\int_0^\tau e^{(\beta-\mu)\tau}d\tau$

$$\therefore \boxed{H = \frac{\alpha_1}{\beta}\left[1-e^{-\beta\tau}\right] + h}$$

$$h = \gamma_1 e^{-\beta\tau} \int_0^{\tau} e^{(\beta-\mu)\tau} d\tau$$

$\boxed{\text{case 1; } \beta = \mu}$

$$h_1(\tau) = \gamma_1 e^{-\beta\tau} \int_0^{\tau} d\tau$$

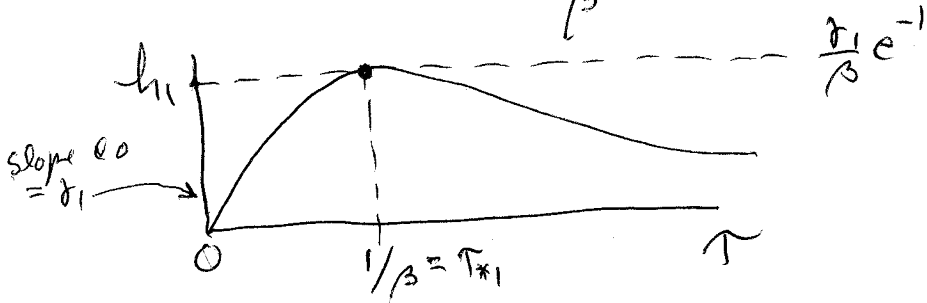
$$\boxed{h_1 = \gamma_1 e^{-\beta\tau}\, \tau}$$

$$\frac{dh_1}{d\tau} = \gamma_1 \left\{ e^{-\beta\tau} + \tau(-\beta) e^{-\beta\tau} \right\}$$

$$= \gamma_1 e^{-\beta\tau} \left\{ 1 - \beta\tau \right\}$$

$$\frac{dh_1}{d\tau} = 0 \quad @ \quad \tau_{*1} = \frac{1}{\beta}$$

$$h_1\left(\tau_{*1} = \frac{1}{\beta}\right) = \frac{\gamma_1}{\beta} e^{-1}$$



$\boxed{\text{case 2; } \beta > \mu}$

$$h_2 = \gamma_1 e^{-\beta\tau} \int_0^{\tau} e^{(\beta-\mu)\tau} d\tau$$

$$h_2 = \frac{\gamma_1 e^{-\beta T}}{\beta - \mu} \int_0^T e^{(\beta-\mu)T} (\beta-\mu)\,dT$$

$$= \frac{\gamma_1 e^{-\beta T}}{\beta - \mu} \left[e^{(\beta-\mu)T} - 1 \right]$$

$$\boxed{h_2 = \frac{\gamma_1}{\beta-\mu} \left[\underbrace{e^{-\mu T}}_{\text{larger}} - \underbrace{e^{-\beta T}}_{\text{smaller}} \right]}$$

$$\frac{dh_2}{dT} = \frac{\gamma_1}{\beta-\mu} \left[-\mu e^{-\mu T} - -\beta e^{-\beta T} \right]$$

$$= \frac{\gamma_1}{\beta-\mu} \left[-\mu e^{-\mu T} + \beta e^{-\beta T} \right]$$

$$\frac{dh_2}{dT} \to 0 \quad @ \quad \beta e^{-\beta T_{*2}} = \mu e^{-\mu T_{*2}}$$

$$\frac{\beta}{\mu} = e^{(\beta-\mu)T_{*2}}$$

$$(\beta-\mu)T_{*2} = \ln(\beta/\mu)$$

$$T_{*2} = \frac{1}{\beta-\mu} \ln\left(\frac{\beta}{\mu}\right)$$

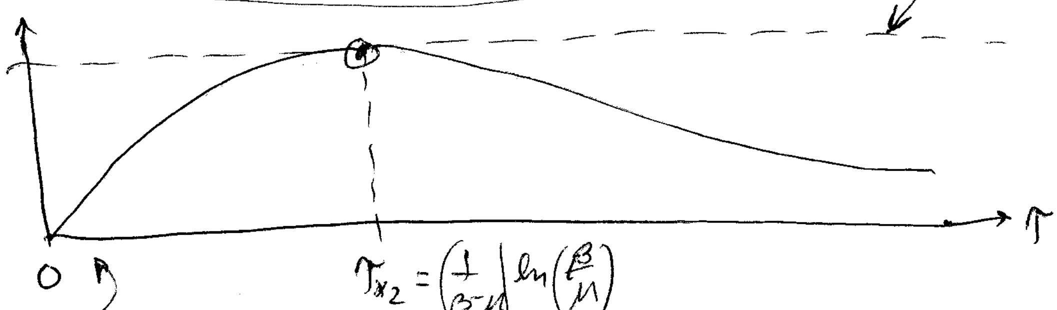
$$h_2(T_{*2}) = \frac{\gamma_1}{\beta-\mu} \left[e^{\frac{-\mu}{\beta-\mu}\ln(\beta/\mu)} - e^{\frac{-\beta}{\beta-\mu}\ln(\beta/\mu)} \right]$$

$$= \frac{\gamma_1}{\beta-\mu} \left[\left(\frac{\beta}{\mu}\right)^{\frac{-\mu}{\beta-\mu}} - \left(\frac{\beta}{\mu}\right)^{\frac{-\beta}{\beta-\mu}} \right]$$

$$h_2(\tau_{*2}) = \frac{\gamma_1}{\beta-\mu}\left[\left(\frac{\mu}{\beta}\right)^{\frac{\mu}{\beta-\mu}} - \left(\frac{\mu}{\beta}\right)^{\frac{\beta}{\beta-\mu}}\right]$$

$$= \frac{\gamma_1}{\beta-\mu}\left(\frac{\mu}{\beta}\right)^{\frac{\mu}{\beta-\mu}}\left[1 - \left(\frac{\mu}{\beta}\right)^{\frac{\beta-\mu}{\beta-\mu}}\right]$$

$$h_2(\tau_{*2}) = \frac{\gamma_1}{\beta-\mu}\left(\frac{\mu}{\beta}\right)^{\frac{\mu}{\beta-\mu}}\left[1 - \frac{\mu}{\beta}\right]; \quad \beta > \mu$$



$\boxed{\text{case 3, } \beta < \mu}$ $\qquad (\mu-\beta) > 0$

$$h_3 = \gamma_1 e^{-\beta\tau}\int_0^{\tau} e^{-(\mu-\beta)\tau}\,d\tau$$

$$= \frac{\gamma_1 e^{-\beta\tau}}{-(\mu-\beta)}\int_0^{\tau} e^{-(\mu-\beta)\tau}\,(-(\mu-\beta))\,d\tau$$

$$= \frac{\gamma_1 e^{-\beta\tau}}{-(\mu-\beta)}\left[e^{-(\mu-\beta)\tau} - 1\right]$$

$$h_3 = \frac{\gamma_1 e^{-\beta\tau}}{(\mu-\beta)}\left[1 - e^{-(\mu-\beta)\tau}\right]$$

$$(\mu-\beta) > 0$$

$$h_3 = \frac{\gamma_1}{\mu-\beta}\Big[\underset{\substack{\uparrow \\ \text{larger}}}{e^{-\beta\tau}} - \underset{\substack{\uparrow \\ \text{smaller}}}{e^{-\mu\tau}}\Big] \qquad \mu > \beta$$

$$\frac{dh_3}{d\tau} = \frac{\gamma_1}{\mu-\beta}\left[-\beta e^{-\beta\tau} - -\mu e^{-\mu\tau}\right]$$

$$= \frac{\gamma_1}{\mu-\beta}\left[-\beta e^{-\beta\tau} + \mu e^{-\mu\tau}\right]$$

$$\frac{dh_3}{d\tau} = 0 \quad @ \quad \tau_{*3} :$$

$$\mu e^{-\mu\tau_{*3}} = \beta e^{-\beta\tau_{*3}}$$

$$\frac{\mu}{\beta} = e^{(\mu-\beta)\tau_{*3}}$$

$$(\mu-\beta)\tau_{*3} = \ln(\mu/\beta)$$

$$\tau_{*3} = \frac{1}{\mu-\beta}\ln\left(\frac{\mu}{\beta}\right)$$

$$h_3(\tau_{*3}) = \frac{\gamma_1}{\mu-\beta}\left[e^{\frac{-\beta}{\mu-\beta}\ln\left(\frac{\mu}{\beta}\right)} - e^{\frac{-\mu}{\mu-\beta}\ln\left(\frac{\mu}{\beta}\right)}\right]$$

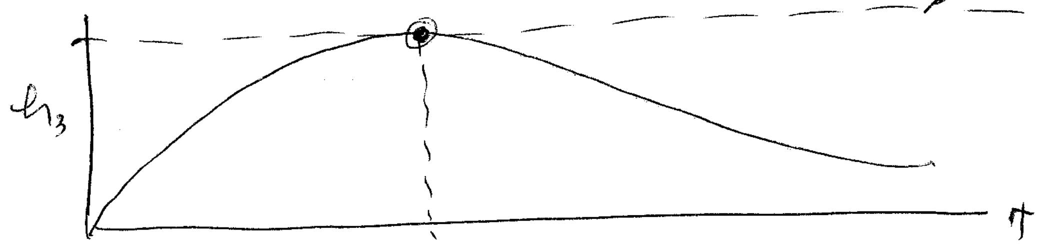
$$= \frac{\gamma_1}{\mu-\beta}\left[\left(\frac{\mu}{\beta}\right)^{\frac{-\beta}{\mu-\beta}} - \left(\frac{\mu}{\beta}\right)^{\frac{-\mu}{\mu-\beta}}\right]$$

$$h_3(\tau_{*3}) = \frac{\gamma_1}{\mu-\beta}\left[\left(\frac{\beta}{\mu}\right)^{\frac{\beta}{\mu-\beta}} - \left(\frac{\beta}{\mu}\right)^{\frac{\mu}{\mu-\beta}}\right]$$

$\boxed{\mu > \beta}$

$$h_3(\tau_{*3}) = \frac{\gamma_1}{\mu-\beta}\left(\frac{\beta}{\mu}\right)^{\frac{\beta}{\mu-\beta}}\left[1 - \left(\frac{\beta}{\mu}\right)^{\frac{\mu-\beta}{\mu-\beta}}\right]$$

$$= \frac{\gamma_1}{\mu-\beta}\left(\frac{\beta}{\mu}\right)^{\frac{\beta}{\mu-\beta}}\left[1 - \frac{\beta}{\mu}\right]$$



$$\tau_{*3} = \frac{1}{\mu-\beta}\ln\left(\frac{\mu}{\beta}\right)$$

slope @ $\tau = 0$
is γ_1

$$H = \frac{\alpha_1}{\beta}\left[1 - e^{-\beta\tau}\right] + h_{1,2,3}$$

$\boxed{\text{for } \beta = \mu}$ $\quad h_1 = \gamma_1 e^{-\beta\tau}\tau$

@ $\tau = \frac{1}{\beta}$; $h = \frac{\gamma_1}{\beta}e^{-1}$, peak h

$$\boxed{H_{\beta=\mu} = \frac{\alpha_1}{\beta}\left[1 - e^{-\beta\tau}\right] + \gamma_1 \tau e^{-\beta\tau}}$$

$\boxed{\text{for } \beta > \mu}$

$$h_{2} = \frac{\gamma_1}{\beta - \mu}\left[e^{-\mu\tau} - e^{-\beta\tau}\right]$$

@ $\tau = \frac{1}{\beta-\mu}\ln\left(\beta/\mu\right)$, h_2 peaks @

$$h_{2*} = \frac{\gamma_1}{\beta-\mu}\left(\frac{\mu}{\beta}\right)^{\frac{\mu}{\beta-\mu}}\left[1 - \frac{\mu}{\beta}\right]$$

$$\boxed{H_{\beta>\mu} = \frac{\alpha_1}{\beta}\left[1 - e^{-\beta\tau}\right] + \left(\frac{\gamma_1}{\beta-\mu}\right)\left[e^{-\mu\tau} - e^{-\beta\tau}\right]}$$

$\boxed{\text{for } \mu > \beta}$

$$h_3 = \frac{\gamma_1}{\mu-\beta}\left[e^{-\beta\tau} - e^{-\mu\tau}\right]$$

@ $\tau = \frac{1}{\mu-\beta}\ln\left(\mu/\beta\right)$, h_3 peaks @

$$h_{3*} = \frac{\gamma_1}{\mu-\beta}\left(\frac{\beta}{\mu}\right)^{\frac{\beta}{\mu-\beta}}\left[1 - \frac{\beta}{\mu}\right]$$

$$\boxed{H_{\mu>\beta} = \frac{\alpha_1}{\beta}\left[1 - e^{-\beta\tau}\right] + \left(\frac{\gamma_1}{\mu-\beta}\right)\left[e^{-\beta\tau} - e^{-\mu\tau}\right]}$$

---

for all cases

$$H(\tau \to \infty) = \frac{\alpha_1}{\beta}$$, asymptotic value

for $t_1 = 137$ (2047) (example #3)

$T_1 = 1.4867\ ^\circ C$

$F_1 = 0.5931$

$A_1 = 0.5226$

$\boxed{\beta = 0.004194\ 1/\text{year}}$ (as before)

$$\boxed{\begin{aligned}\sigma &= 3.101 \times 10^{-8}\ ^\circ C/s \\ &= 0.9786\ ^\circ C/yr \\ \chi &= 3.483 \times 10^{-8}\ ^\circ C/s \\ &= 1.0992\ ^\circ C/yr\end{aligned}}$$

$\alpha_1 = \{\sigma(1-A_1) - \chi(1-F_0)\}$; $\boxed{F_0 = 0.3769}$

$$\boxed{\alpha_1 = -0.2177\ ^\circ C/yr}$$

$\gamma_1 = \chi[F_1 - F_0] = 0.2376\ ^\circ C/yr$

$$\boxed{\gamma_1 = +0.2376\ ^\circ C/yr}$$

$$\therefore \boxed{\frac{\alpha_1}{\beta} = -51.9075\ ^\circ C}$$

$$H = -51.9075\ ^\circ C\left[1 - e^{-0.004194\tau}\right] + \left(+0.2376\ \frac{^\circ C}{yr}\right)\tau\, e^{-0.004194\tau}$$

, for $\mu = \beta$

$\beta = 0.004194$

$$H = -51.9075\,°C\left\{1 - e^{-0.004194\tau}\right] + \left(\frac{0.2376}{0.004194 - \mu}\right)\left[e^{-\mu\tau} - e^{-0.004194\tau}\right], \quad \beta > \mu$$

long residence time

$$H = -51.9075\,°C\left[1 - e^{-0.004194\tau}\right] + \left(\frac{0.2376}{\mu - 0.004194}\right)\left[e^{-0.004194\tau} - e^{-\mu\tau}\right], \quad \mu > \beta$$

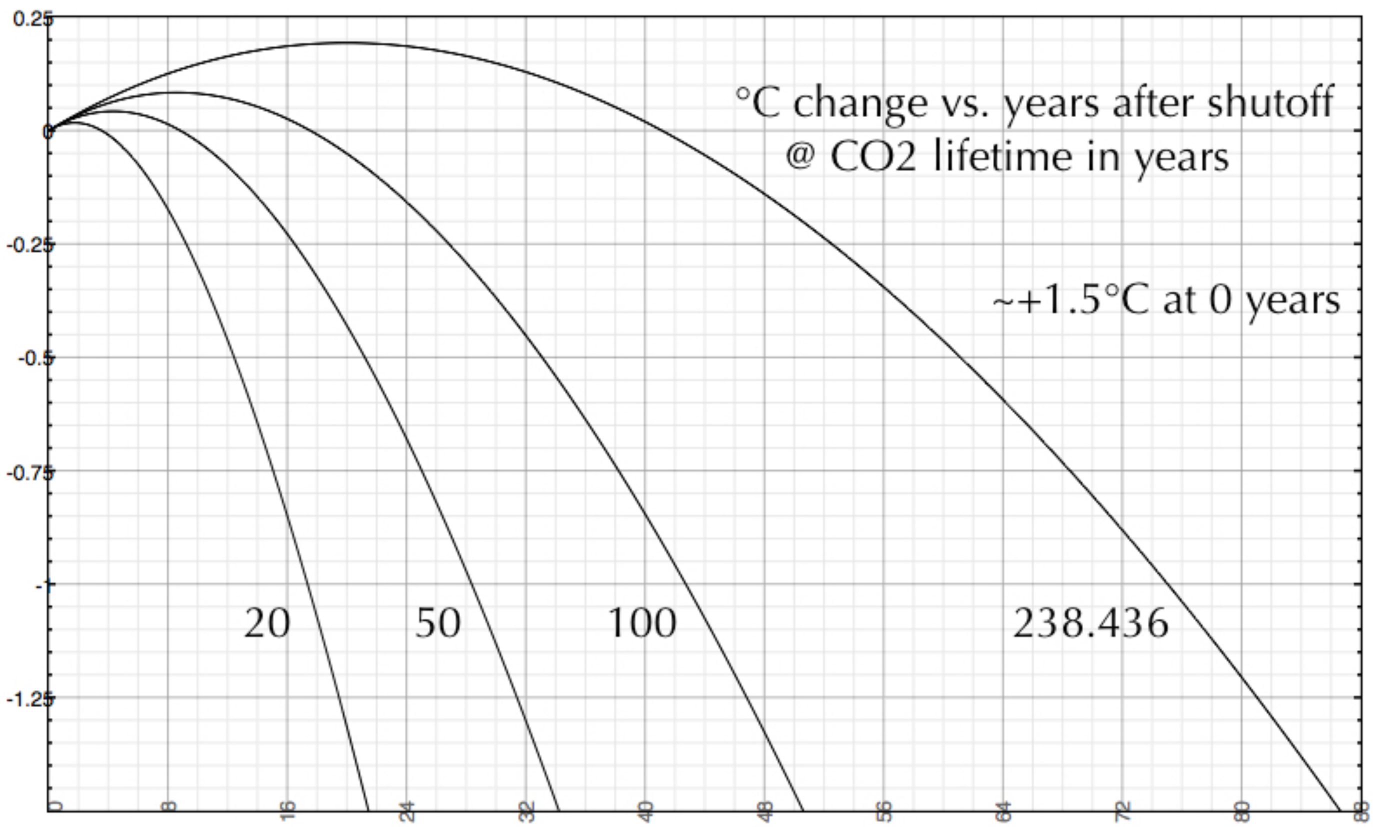
short residence time

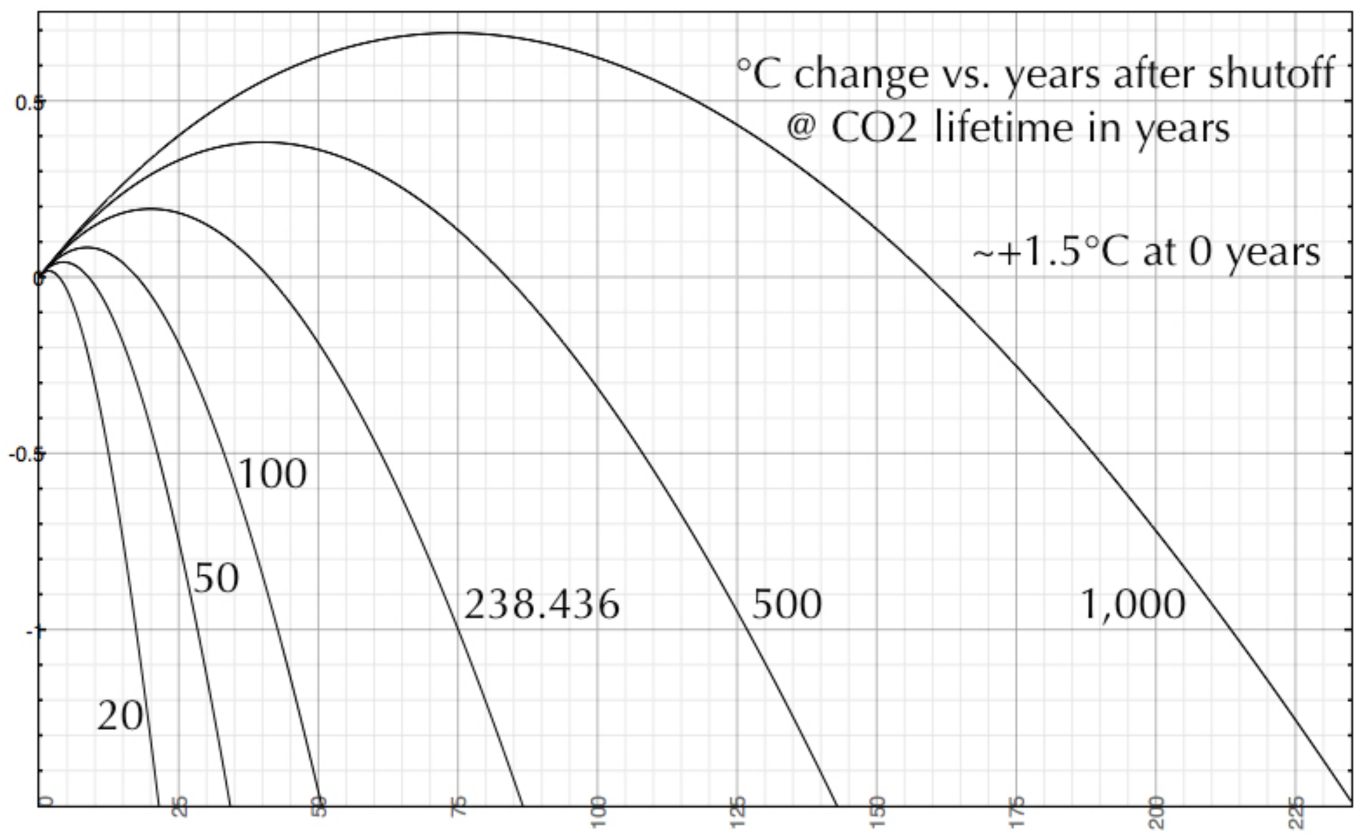
$$\frac{1}{\beta} = 238.436 \text{ years}$$

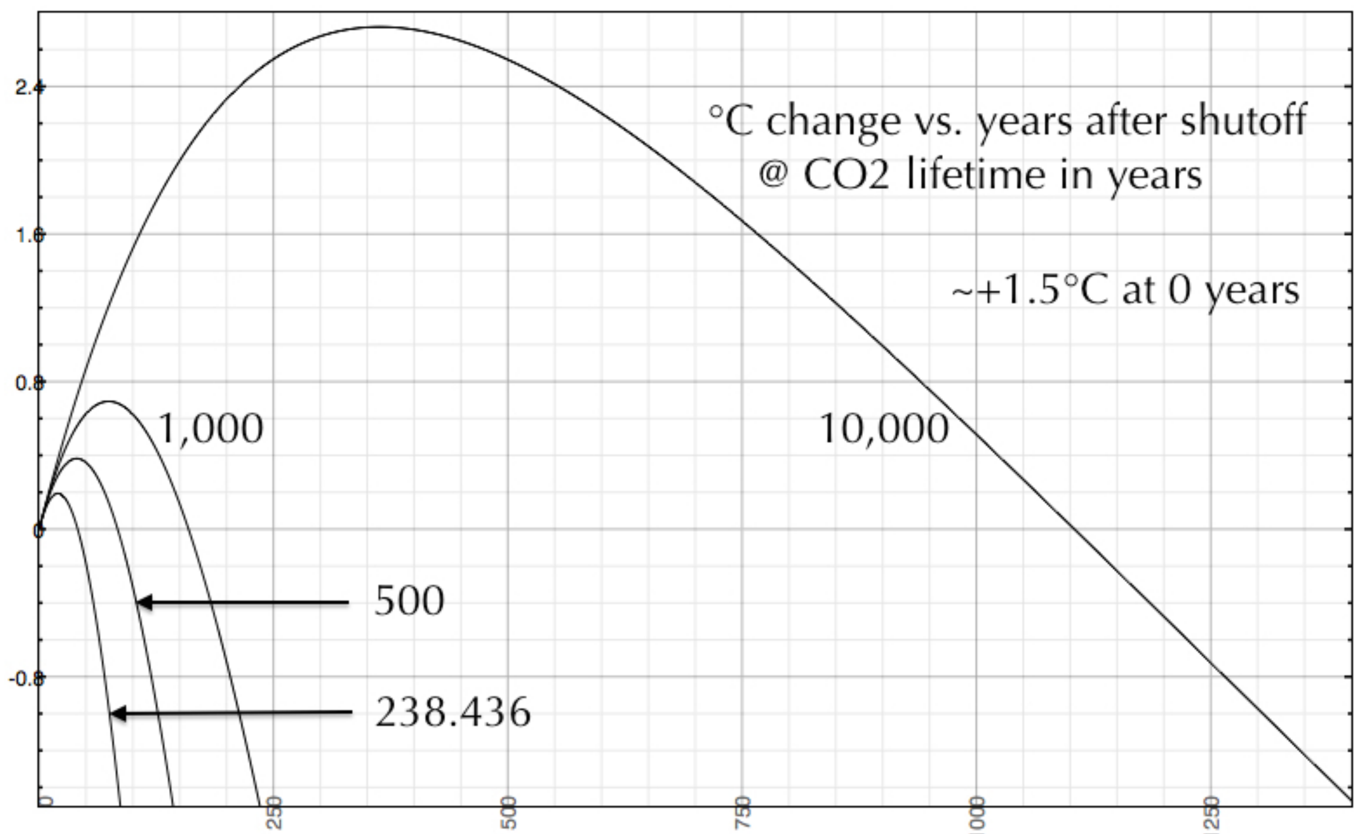
pick $\mu = \frac{1}{20}, \frac{1}{50}, \frac{1}{100}, \beta = \mu = \frac{1}{238.436}, \frac{1}{500}, \frac{1}{1000}$

$\mu = \frac{1}{10{,}000}, \frac{1}{100{,}000}$

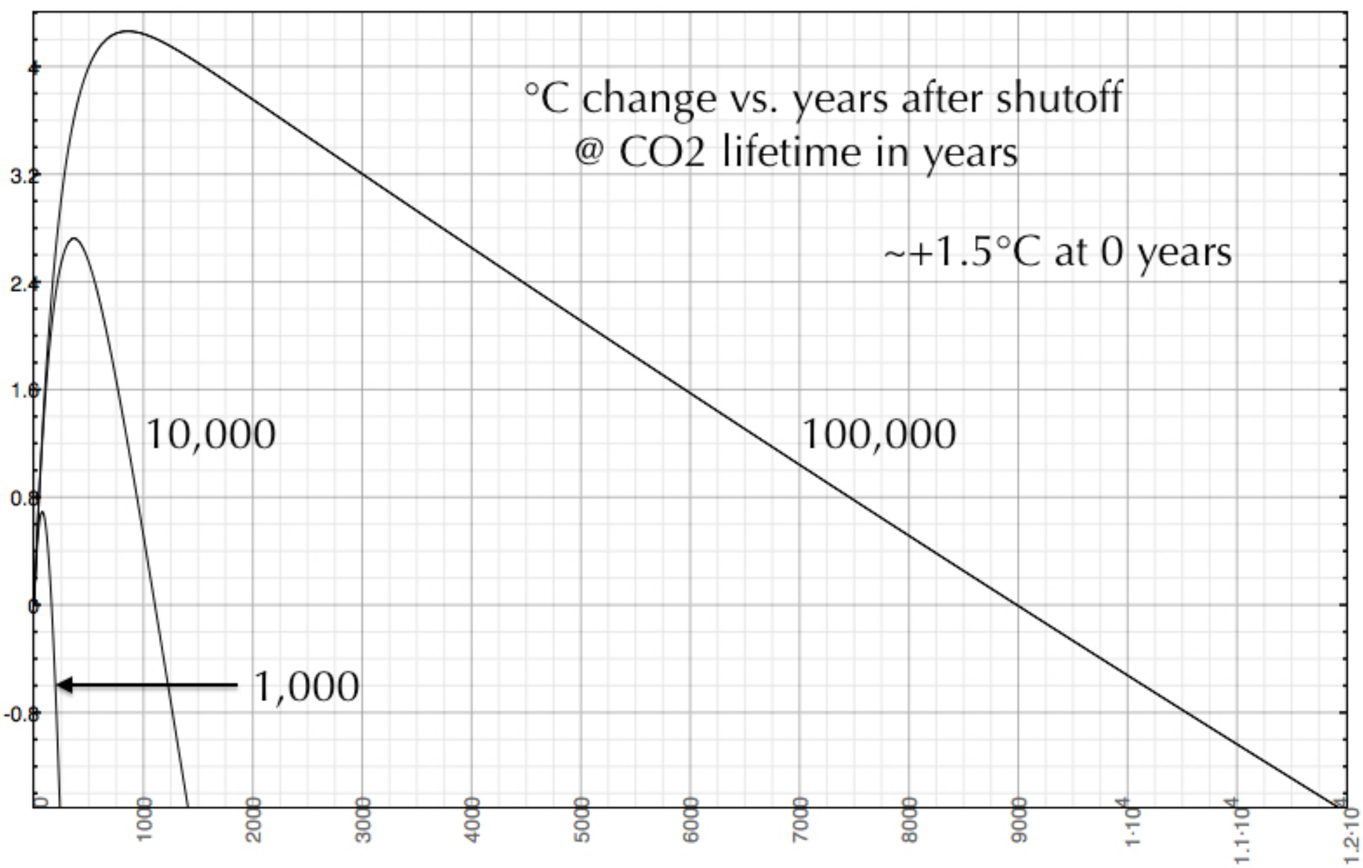
eight cases


°C change vs. years after shutoff
@ CO2 lifetime in years
~+1.5°C at 0 years
20
50
100
238.436
0.25
0
-0.25
-0.5
-0.75
-1
-1.25
0
8
16
24
32
40
48
56
64
72
80
88


°C change vs. years after shutoff
@ CO2 lifetime in years
~+1.5°C at 0 years
0.5
0
-0.5
-1
20
50
100
238.436
500
1,000
0
25
50
75
100
125
150
175
200
225



°C change vs. years after shutoff
@ CO2 lifetime in years
~+1.5°C at 0 years
1,000
10,000
500
238.436
2.4
1.6
0.8
0
-0.8
0
250
500
750
1000
1250



°C change vs. years after shutoff
@ CO2 lifetime in years
~+1.5°C at 0 years
10,000
100,000
1,000
4
3.2
2.4
1.6
0.8
0
-0.8
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
1·10^4
1.1·10^4
1.2·10^4